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Shaping Future of Wireless: Two-way Connectivity

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Outline

1. Introduction

2. Basics of RF Design and Signal Processing

- ✧ Literature survey and comparisons

3. Applications

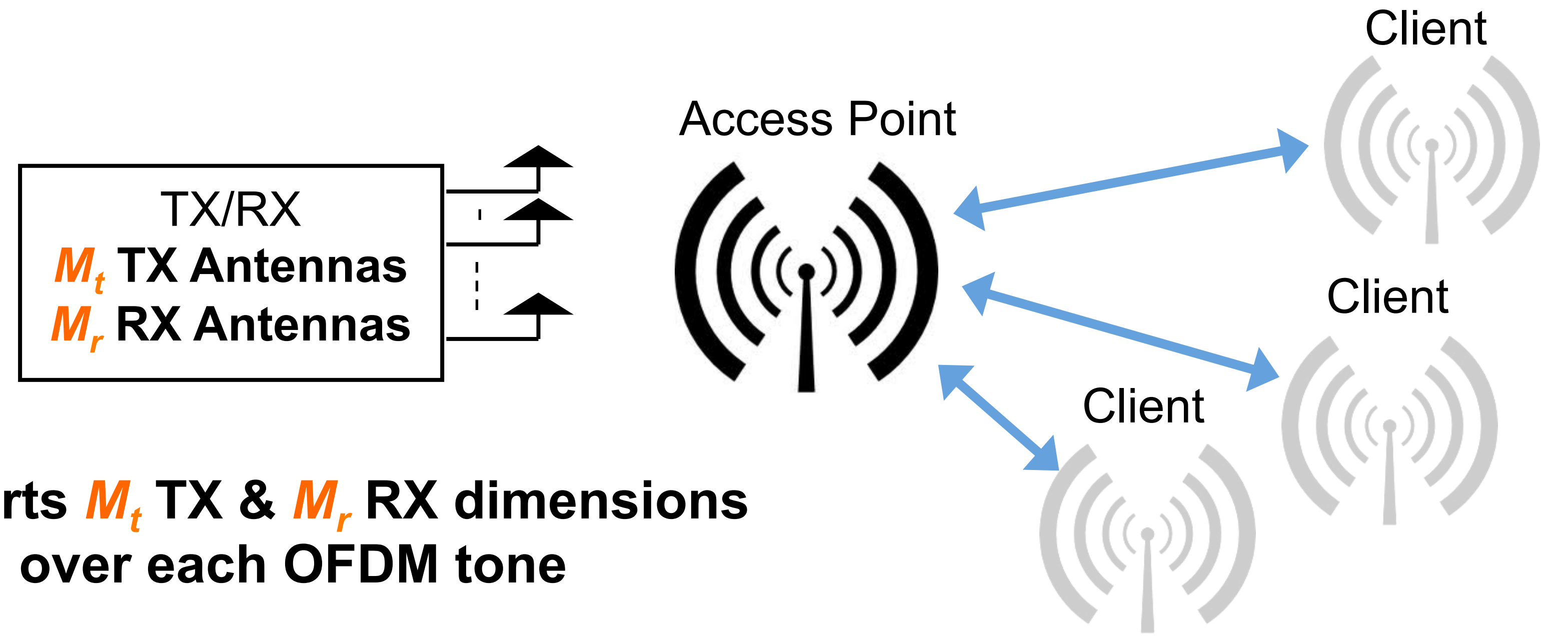
- ✧ Network Applications
- ✧ A New Paradigm: Media-based Wireless
- ✧ Security Applications

4. Conclusion & Acknowledgements

This talk focuses on main points. Details, including video clips showing real-time performance are at:

www.cst.uwaterloo.ca/2way

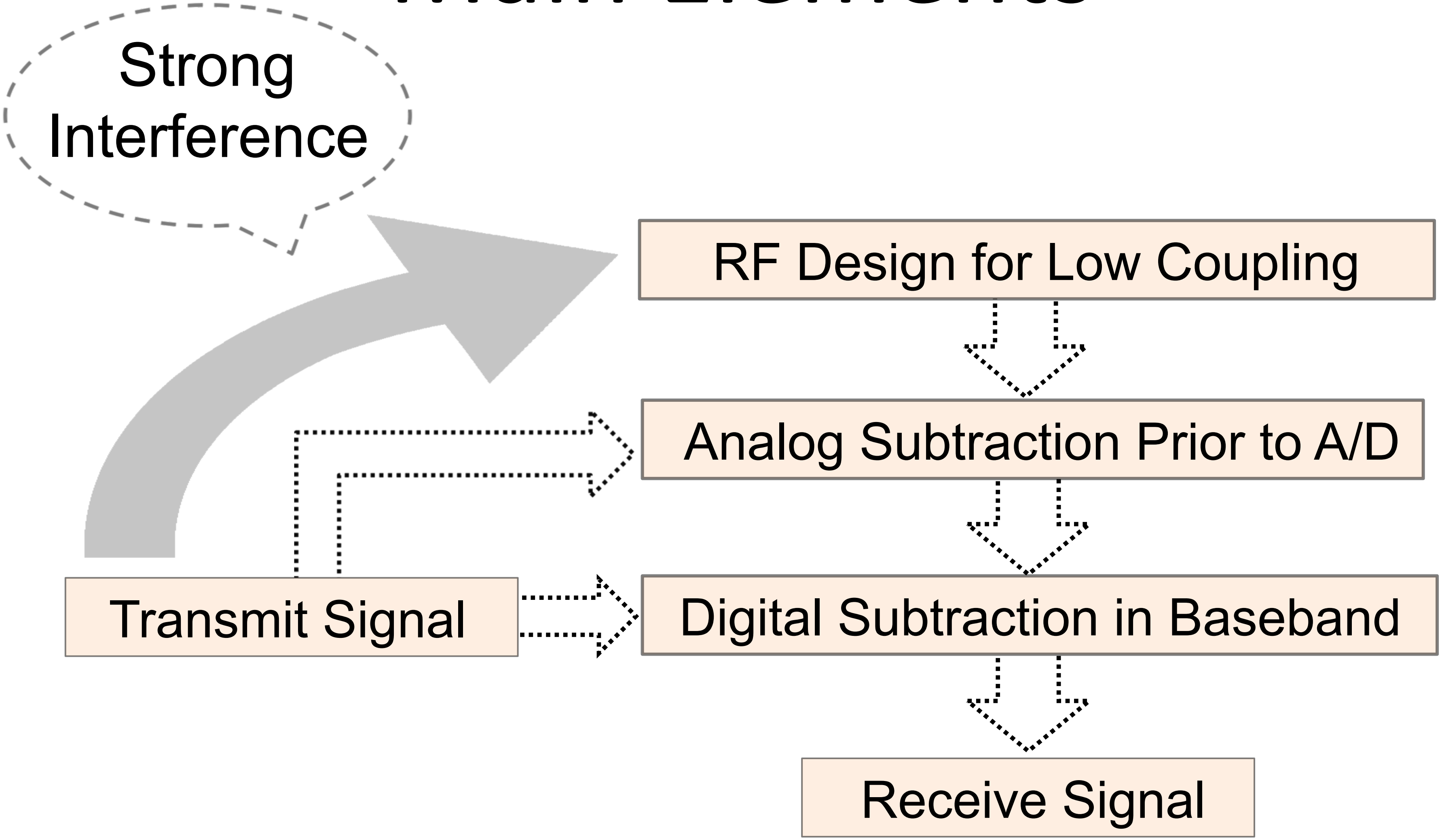
Basic Setup (User Multiplexing)



Supports M_t TX & M_r RX dimensions over each OFDM tone

- Divide OFDM tones among several clients (i.e., OFDMA) with two-way communications over each tone
  - Simple/small antennas, simple signal processing, off-the-shelf hardware
  - Support for MIMO in each tone
  - Clients do not need to be synchronized
  - **Clients do not require to have full-duplex capability**

Main Elements



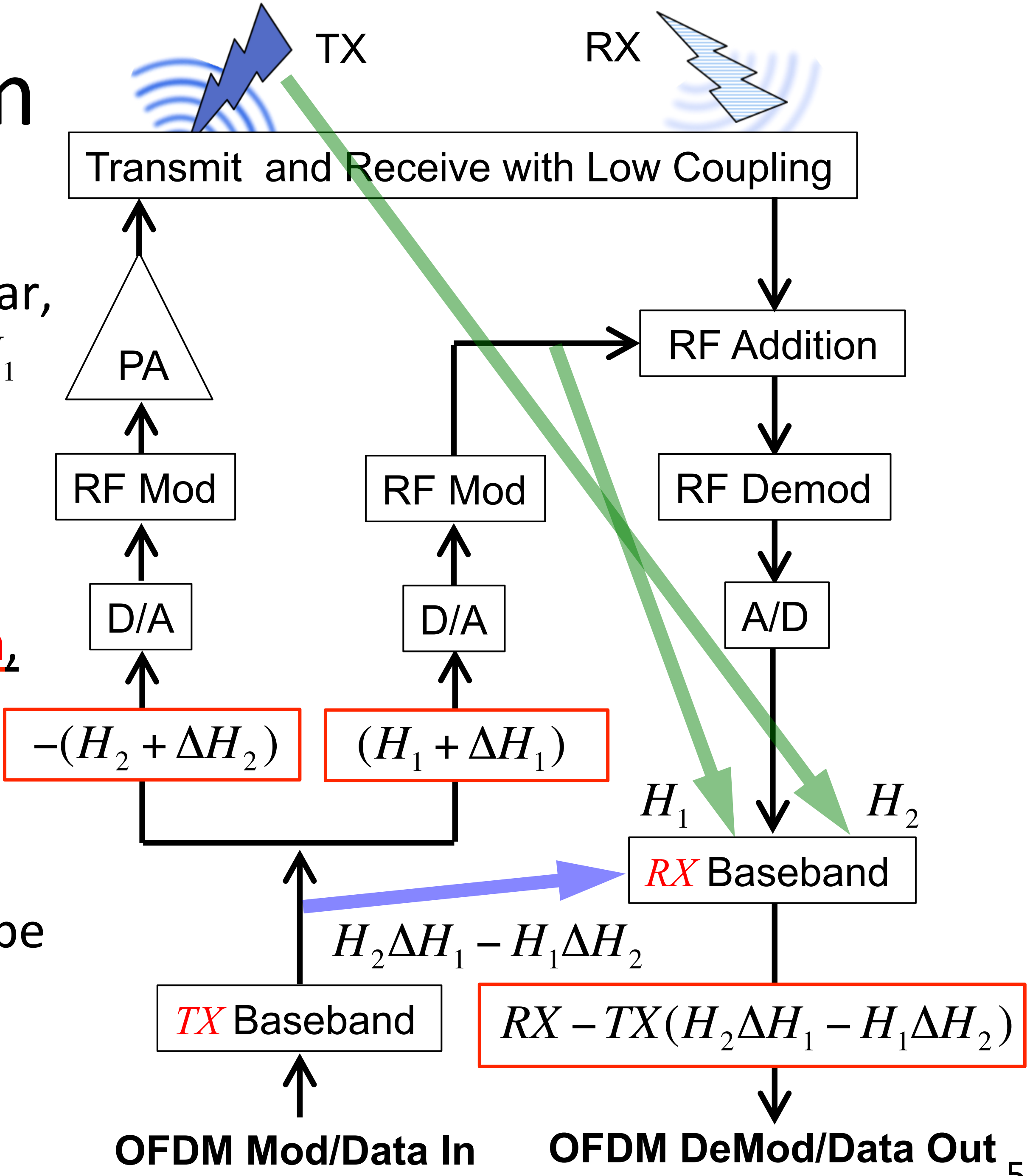
Block Diagram

Critical Point: D/A is linear, as a result $H_2\Delta H_1 - H_2\Delta H_1$ can be measured.

Procedure: Not to worry too much about error terms $\Delta H_1, \Delta H_2$, **fix them**, and then measure:

$$H_2\Delta H_1 - H_2\Delta H_1$$

As $H_2\Delta H_1 - H_2\Delta H_1$ is of lower magnitude, it can be measured accurately.



TX

RX

Transmit and Receive with Low Coupling

PA

RF Addition

RF Mod

RF Mod

RF Demod

D/A

D/A

A/D

$-(H_2+\Delta H_2)$

$(H_1+\Delta H_1)$

H_1

H_2

RX Baseband

$H_2\Delta H_1 - H_1\Delta H_2$

TX Baseband

$RX - TX(H_2\Delta H_1 - H_1\Delta H_2)$

OFDM Mod/Data In

OFDM DeMod/Data Out

What is New and Why it Works?

1 Antennas are designed to reduce coupling, in theory to zero, in practice to -70 to -30dB **(new)**.

2 Analog active cancellation prior to A/D relying on:

- **KEY POINT:** D/A operation is linear (unlike A/D). Form an analog signal to subtract in the receive chain prior to A/D, **while maintaining linearity (new).**

3 Digital active cancellation after A/D.

- Parallel and independent work by Rice University.
- Rice University's approach is not effective due to missing the **KEY POINT** mentioned above. As discussed in their papers, digital cancellation when combined with analog, can degrade the performance of the analog cancellation stage. This is in contrast to the speaker's approach.

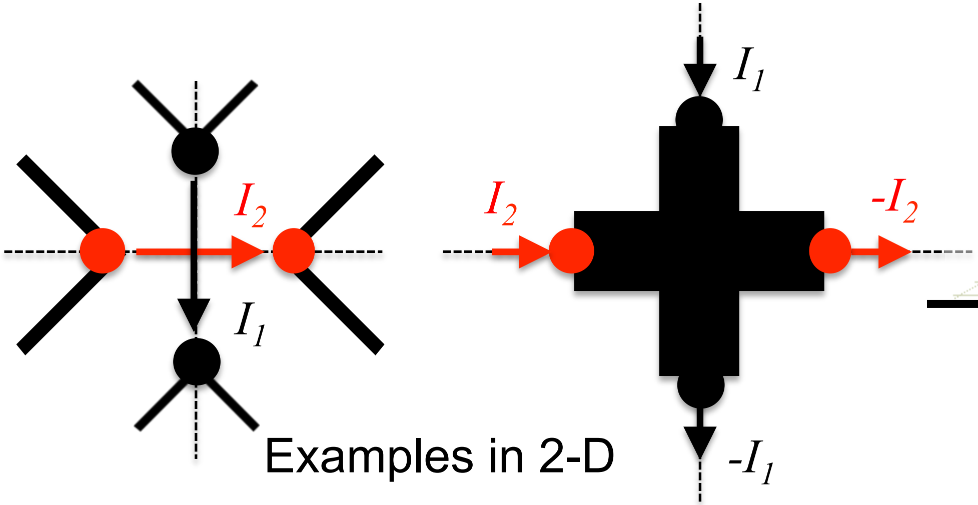
RF Design: Starting Point

- Near field works to our advantage:
  - Strong but predictable, and as a result, totally manageable
- Maxwell equations indicate:
  - Linearity
  - **Geometrical symmetry** in:
    - Construct (shape, material, boundary conditions), and
    - Excitation (antenna feed terminals)

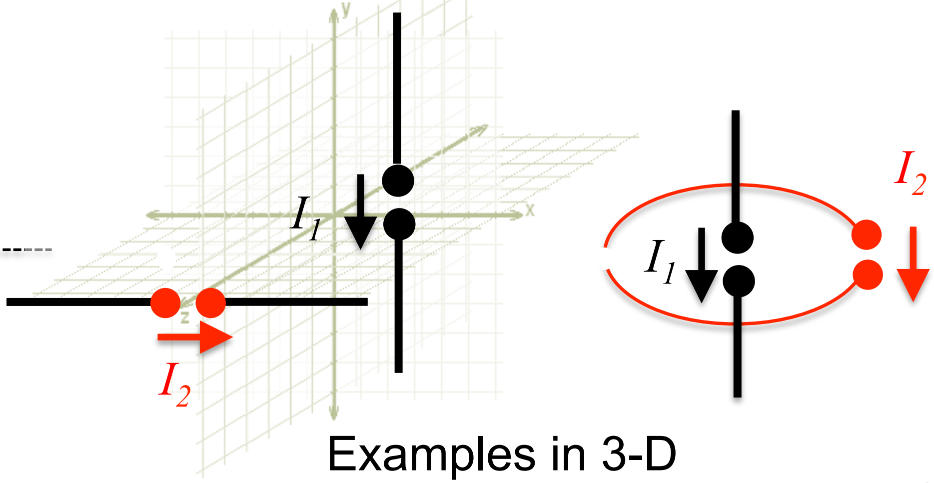
    cause **geometrical symmetry** in wave.
  - Symmetry in wave can be used to cancel signals.

Pair-wise Symmetrical Antennas

- Each antenna is ***Self Symmetrical***:
  - Two arms are image of each other with respect to a plane of symmetry (construct & excitation).
  - Note that arms can overlap (applicable to patch structures).
- Two antennas are ***Mutually Symmetrical***:
  - Antennas have separate planes of symmetry, and are invariant under reflection in the plane of symmetry of each other.



Examples in 2-D



Examples in 3-D

Effect of Symmetry

- For pair-wise symmetrical TX/RX, we have:

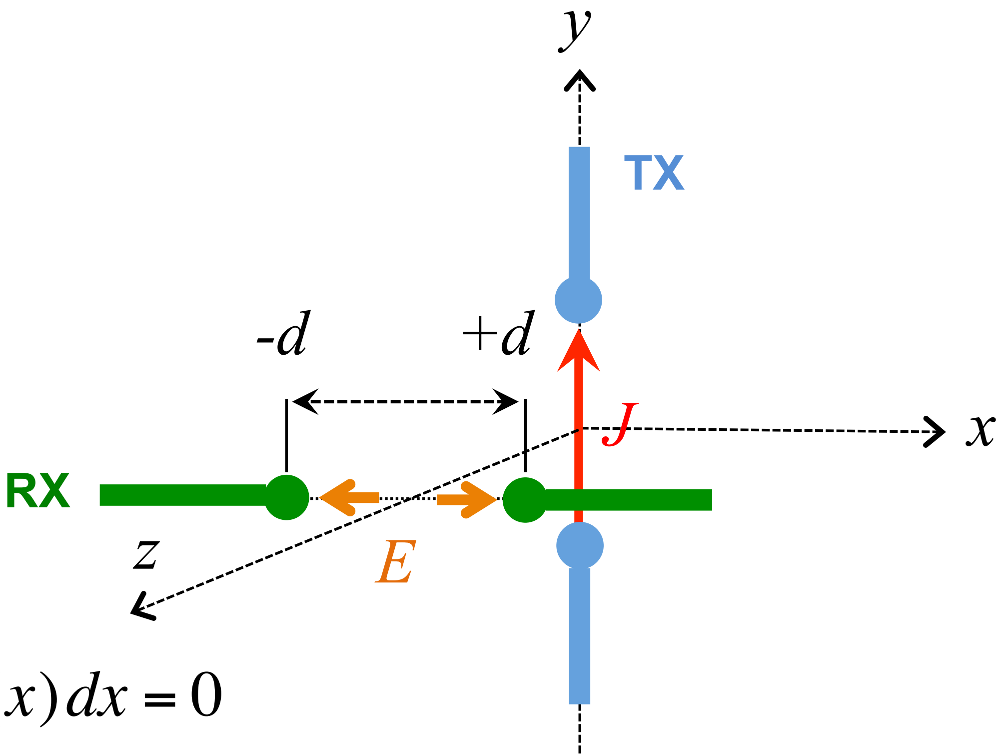
Theorem 2: $S_{12}= S_{21}= 0$ independent of frequency.

Proof: Follows relying on *Theorem* 1 and integrating *E*-field over the line connecting the terminal of the RX antenna.

$x \longrightarrow -x$ Does not Flips J

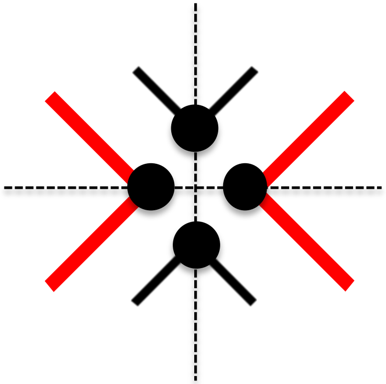
$y \longrightarrow -y$ Flips J

$z \longrightarrow -z$ Does not flip J

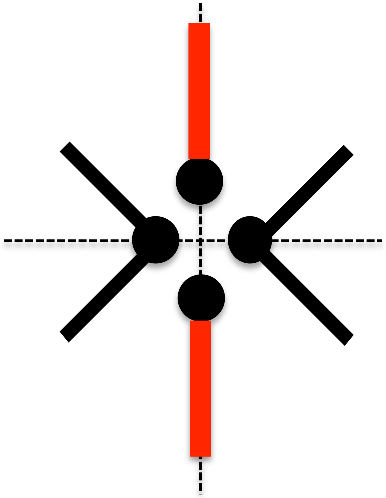


$$E(x) = -E(-x) \Rightarrow \Delta V = \int_{-d}^{+d} E(x)\,dx = 0$$

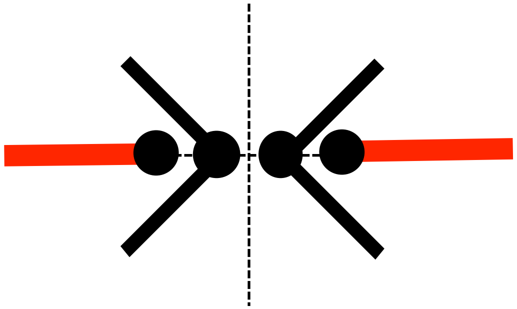
Numerical Results Using HFSS



$S_{12} \approx -90\text{dB}$



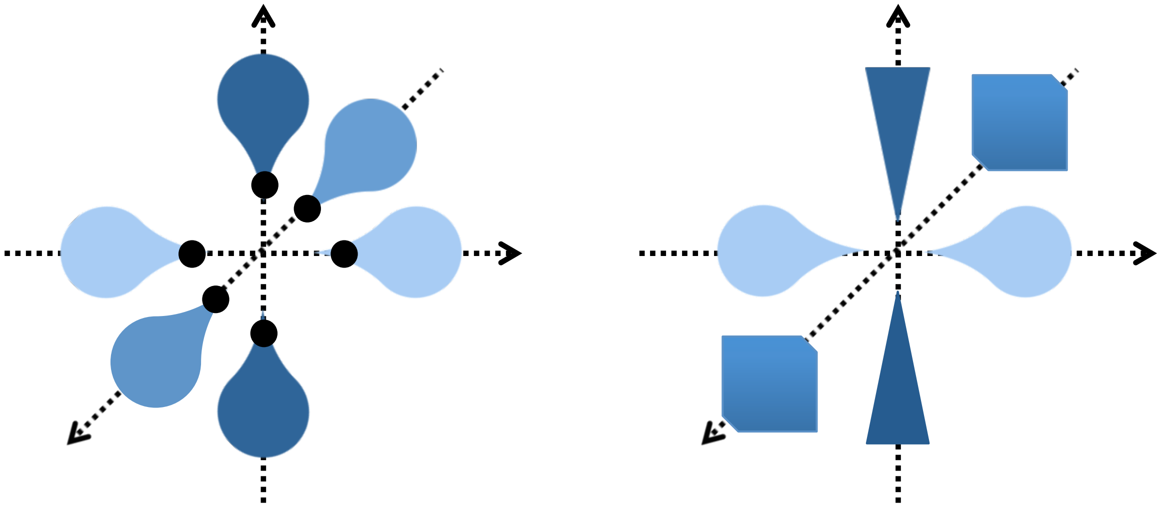
$S_{12} \approx -90\text{dB}$



$S_{12} \approx -2\text{dB}$

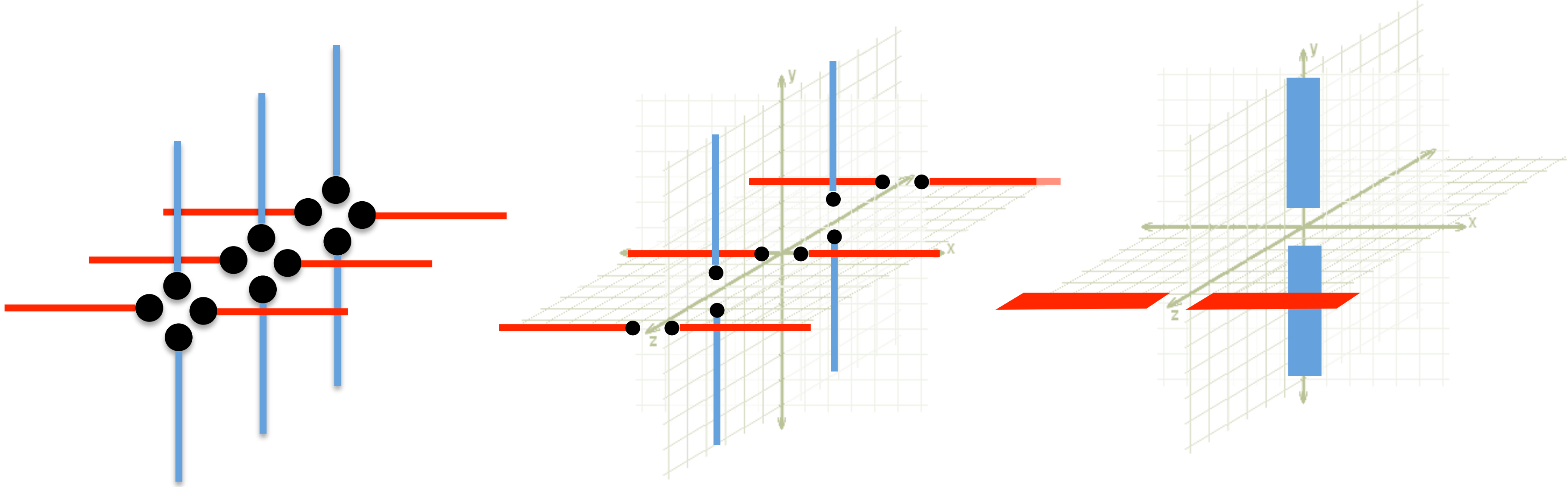
Triple-wise Symmetrical Antennas

- Any two antennas are pair-wise symmetrical; no coupling between any pair.
  - Diagonal S-matrix (independent of frequency)
  - Each antenna can switch between TX and RX modes asynchronous of other ones.



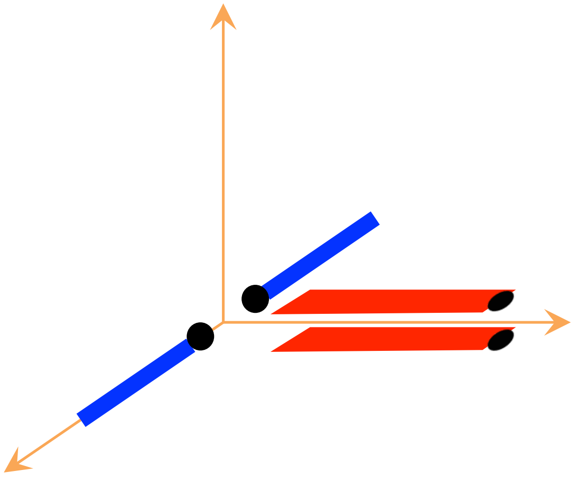
MIMO in 3-D (Zero Coupling)

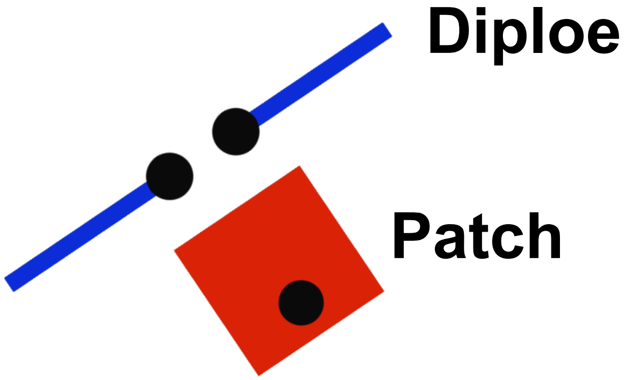
- More flexible (easier to implement) due to the possibility of exploiting three dimensions.



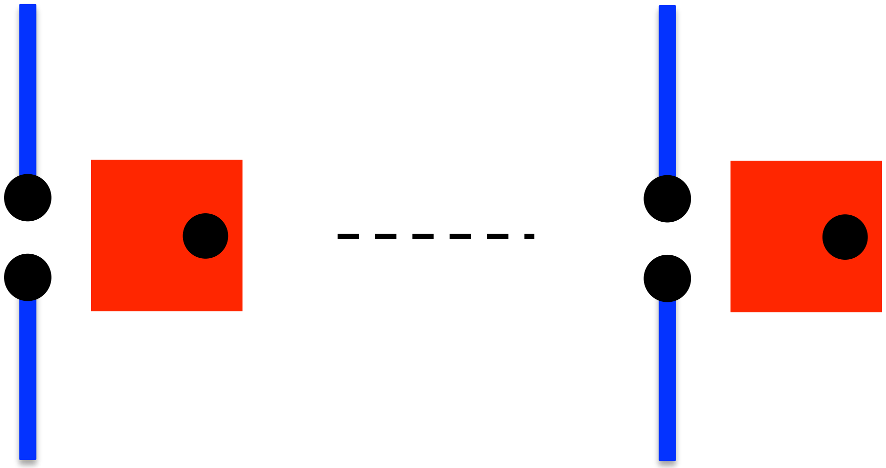
Antennas can be placed in any order, and can be of different lengths for multi-band operation.

Another Form of Decoupled Antennas in 3-D (reality of 2-D)



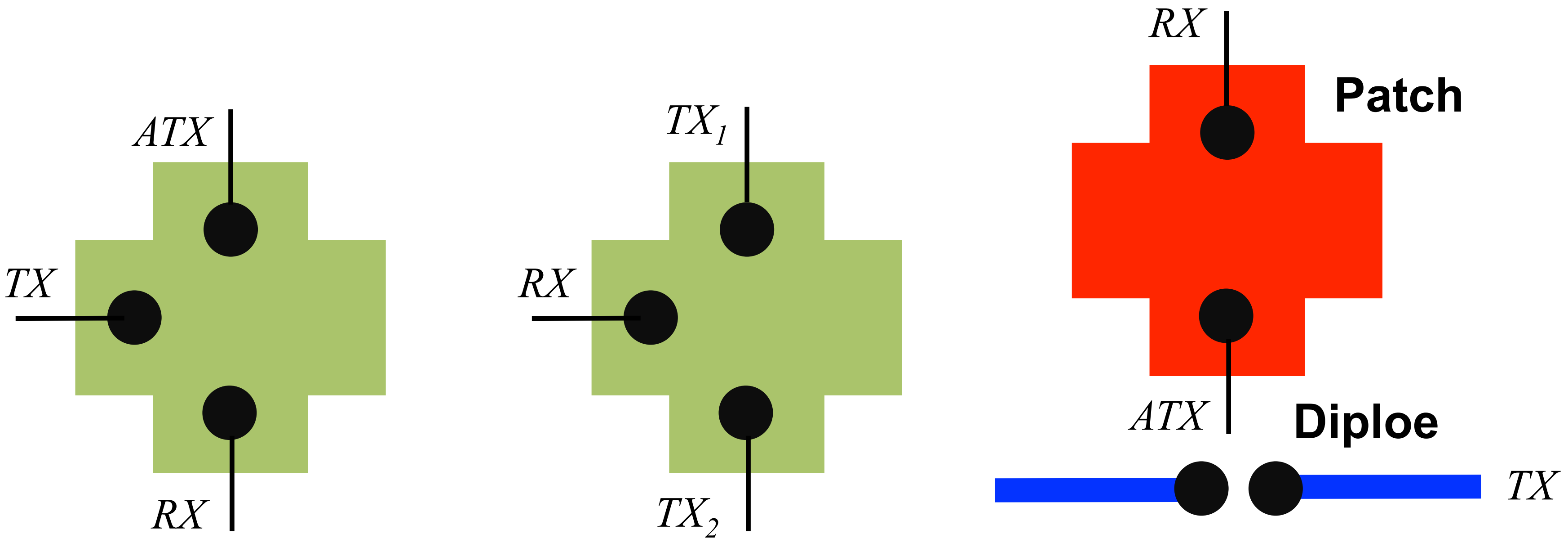


Generalization to MIMO



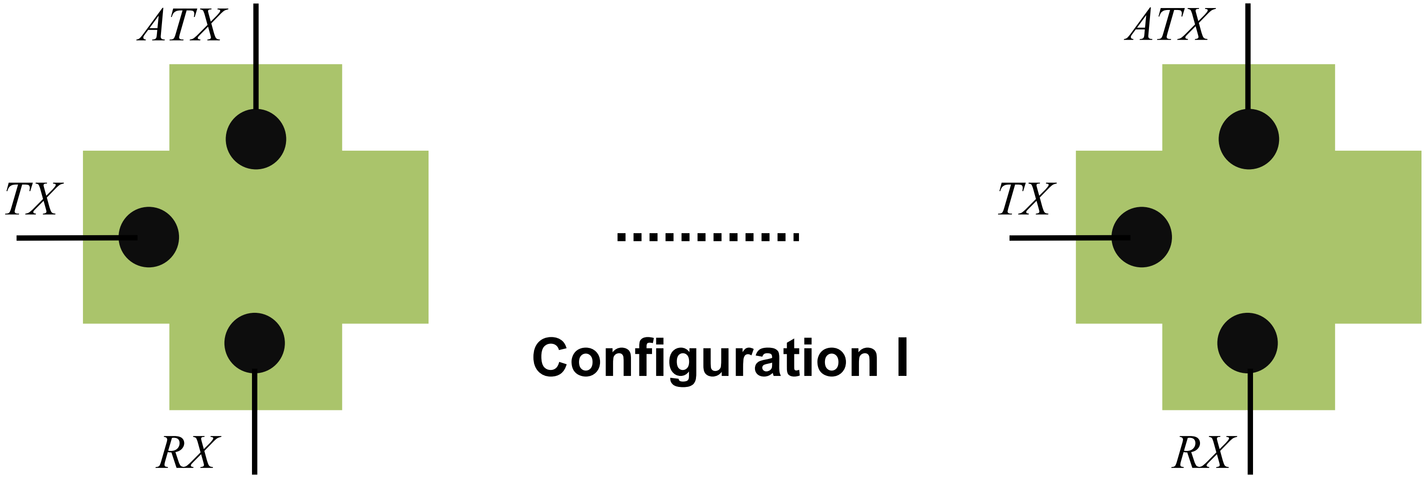
Combined Design: Multi-terminal Antennas

- Transmit, receive and active cancellation are combined into a single arm above ground using patch antennas.

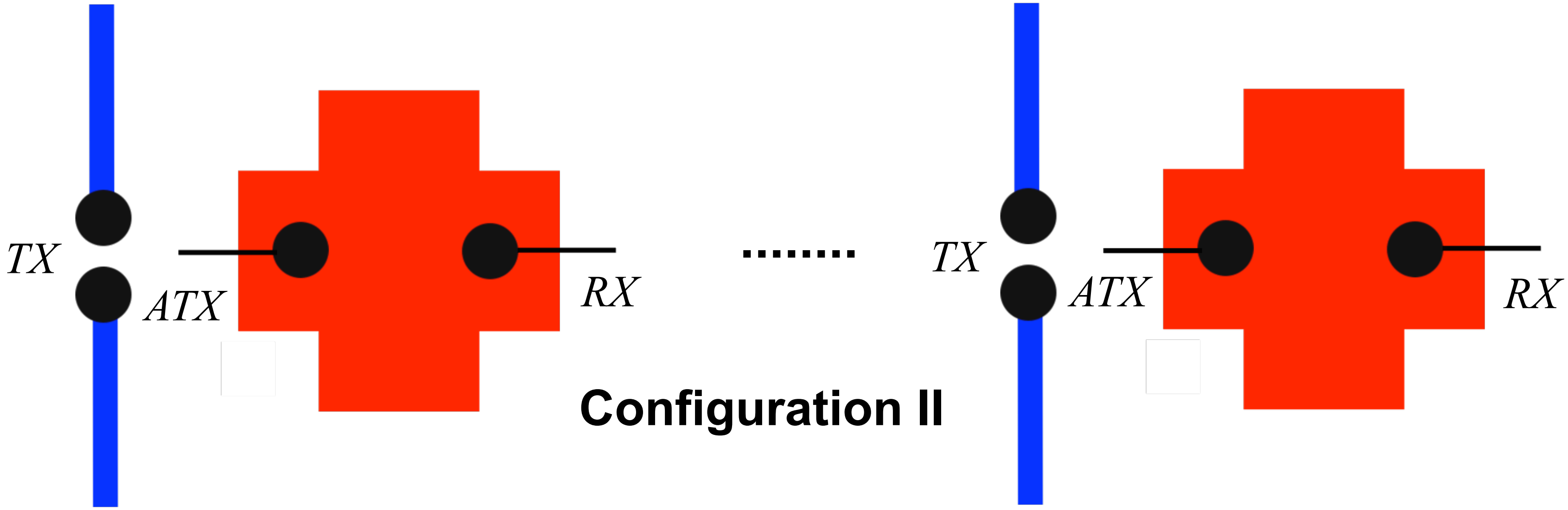


Shape of corner cuts and various sizes are optimized for best radiation efficiency and proper coupling.

Generalization to MIMO: Each Radio has Shared TX/RX Antennas



Configuration I



Configuration II

Example of Performance

- Transmit power about 30dBm (typical for cell phone).
- Residual Self Interference to Noise Ratio:
  - Antenna structure alone: **about 40dB**
  - After analog cancellation: **about 2dB**
  - After digital cancellation: **about 0.4dB**
    - **NOTE**: Observed degradation is less than typical degradations due to various mismatches in signaling between **separate** (distant) TX/RX units.

A Brief Literature Survey

1. K. Tsubouchi, H. Nakase, A. Namba, K. Masu, “Full duplex transmission operation of a 2.45-GHz asynchronous spread spectrum using a SAW convolver”, *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, **Sept. 1993** (Res. Inst. of Electr. Commun., Tohoku Univ., Japan)
   - ➢ Uses different CDMA codes in each directions, single antenna
2. S. Chen, M. A. Beach, J.P. McGeehan, “Division-free duplex for wireless applications,” Electronics Letters, **Jan. 1998**, (Centre for Commun. Res., Bristol Univ.)
   - ➢ First true full duplex, separate TX and RX antennas
3. Amir K. Khandani, “Methods for spatial multiplexing of wireless two-way channels,” US patent, filed Oct. 2006 (provisional patent filed **Oct. 2005**), issued **Oct. 2010**
   - ➢ Analog cancellation using multiple antennas
4. B. Radunovic, D. Gunawardena, P. Key, A. Proutiere, N. Singh, V. Balan, G. Dejean, “Rethinking Indoor Wireless: Low Power, Low Frequency, Full-duplex, "Microsoft Technical Report MSR-TR-2009-148", **July 2009**
   - ➢ Analog cancellation using noise cancelling chip Quellan QHx220
5. M. Duarte and A. Sabharwal, “Full-Duplex Wireless Communications Using Off-The-Shelf Radios: Feasibility and First Results”, *Asilomar Conference on Signals, Systems, and Computers,* **Nov. 2010** (plus three more references from the same team to follow)
   - ➢ Analog active cancellation
6. J. Choiy, M. Jainy, K. Srinivasany, P. Levis, S. Katti, "Achieving Single Channel, Full Duplex Wireless Communication," *Mobicom 20*10, **Sept. 2010**
   - ➢ Based on antenna setup first introduced in speaker’s patent above - Analog cancellation using noise cancelling chip Quellan QHx220
7. Announcement by Stanford University **(Feb. 2011)**: http://www.youtube.com/watch?v=RiQb5NdDWgk
   - ➢ Based on the setup in reference 6 above and reference 11 (see next page)
8. Announcements by Rice University **(Sept. 2011)**: http://www.youtube.com/watch?v=tXMwn2mm0VY
   - ➢ Based on the setup in reference 5 above and references 9,10,12 (see next page)

A Brief Literature Survey (cont.)

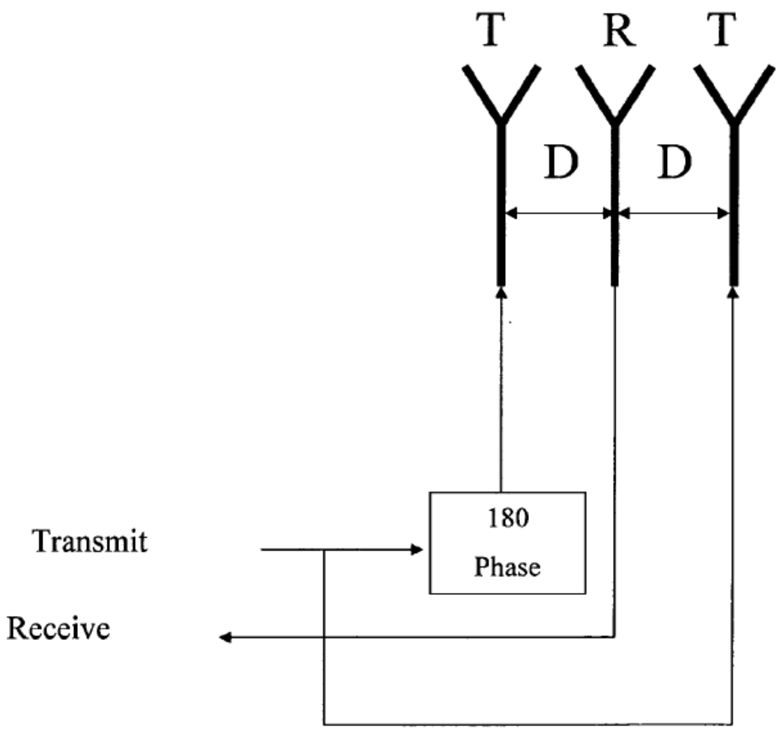
9. A. Sahai, G. Patel and A. Sabharwal, "Pushing the Limits of Full-duplex: Design and Real-time Implementation", *Rice tech report,* **Feb. 2011**
10. M. Duarte, C. Dick and A. Sabharwal, "Experimental-driven Characterization of, “Full-Duplex Wireless Systems", submitted to IEEE Transactions on Wireless Communications, **June 2011**
11. M. Jain, J. Choi, T. Kim, D. Bharadia, S. Seth, K. Srinivasan, P. Levis, S. Katti, P. Sinha, "Practical, Real-time, Full-Duplex Wireless", *Mobicom,* **Sept. 2011**
12. E. Everett, M. Duarte, C. Dick, and A. Sabharwal, "Exploiting Directional Diversity in Full-duplex Communications", *Asilomar Conference,* **Nov. 2011**

- ***300+ more references (articles and patents) on full duplex wireless, active cancellation, etc., with no overlap with the work presented here.***

History of this Work

- Basic hardware implementation functional in late 2009
- Avoid publicizing due to:
  - Aiming for a stronger impact by a more mature system
    - Earlier designs starting from 90's were forgotten as they did not meet industrial standards for practical deployment.
    - All reported designs have a limited functionality in terms of error performance, band-width, etc., and need a controlled/ laboratory environment to establish even the basic connection.
    - IP protection of different aspects
- Why Now?
  - It is mature:
    - Cannot be moved forward (at least not fast enough) without industry's involvement and wider academic research.
  - Academic duty:
    - Knowledge sharing to avoid rediscoveries.

What is New w.r.t. Speaker's Issued Patent?



- New methods for antenna design
- New RF and base-band processing brings degradation in SNR due to self-interference close to zero.
- Support for asynchronous clients (superimposed networking)
- Support for MIMO
- New applications for full-duplex wireless
- Hardware, RF and DSP complexities are virtually the same as half-duplex units.

Figure extracted from speaker's patent issued in 2010 – same antenna setup reported in references 6 (Sept 2010) and 7 (Feb. 2011)

United States Patent **Patent No.: US 7,817,641 B1**
Khandani **Date of Patent: Oct. 19, 2010**

Comparison with RICE and Stanford Universities' Designs

- Both reported in 2010-2011.
- Both are reported under test in restricted environments, while our case is very robust to the operating conditions.
- Ours has much better performance in SNR, smaller size, lower complexity, no band-width limitations.
- Antenna design:
  - Rice Team: Do not get into antenna design.
  - Stanford Team (2011): Use an antenna design which is in my issued patent (filed 2006, issued 2010). What is reported today is way beyond my 2006 patent.

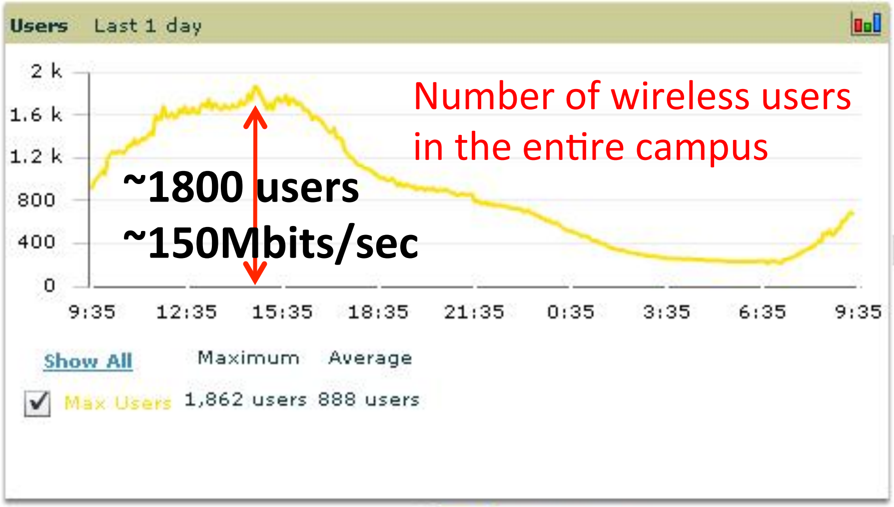
Comparison with RICE and Stanford Universities' Designs

- Analog/digital cancellation:
  - Rice Team: No analog cancellation due to antenna structure, do not rely on the linearity issues mentioned earlier, as a result, digital cancellation can even degrade their performance when combined with analog active cancellation (as they have mentioned in their papers), and this never happens in our case.
  - Stanford Team: Analog cancellation is based on some simple adaptive RF filters. Digital cancellation is a simple subtraction and (same as Rice team's) do not rely on the linearity issues mentioned earlier.

Network Applications

WLAN Traffic: Bursty, Delay Sensitive & Inefficient

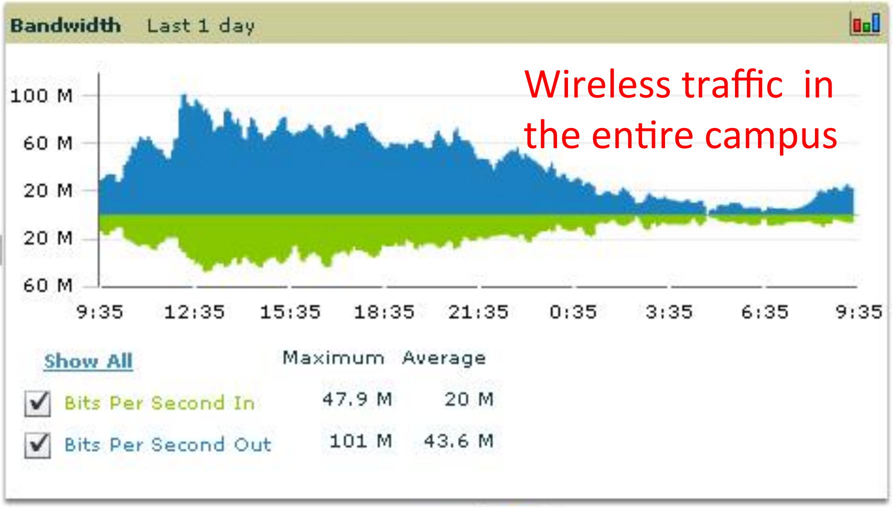
- UW Campus: Total of 1400 wireless APs across campus serving 4200 active wireless IP
  - In theory, each AP should support at least around 54Mbits/sec
  - They usually install an additional AP if the typical number of clients connecting to an AP exceeds 10 most of the time



	Maximum	Average
☑ Max Users	1,862 users	888 users

Number of wireless users in the entire campus

~1800 users
~150Mbits/sec

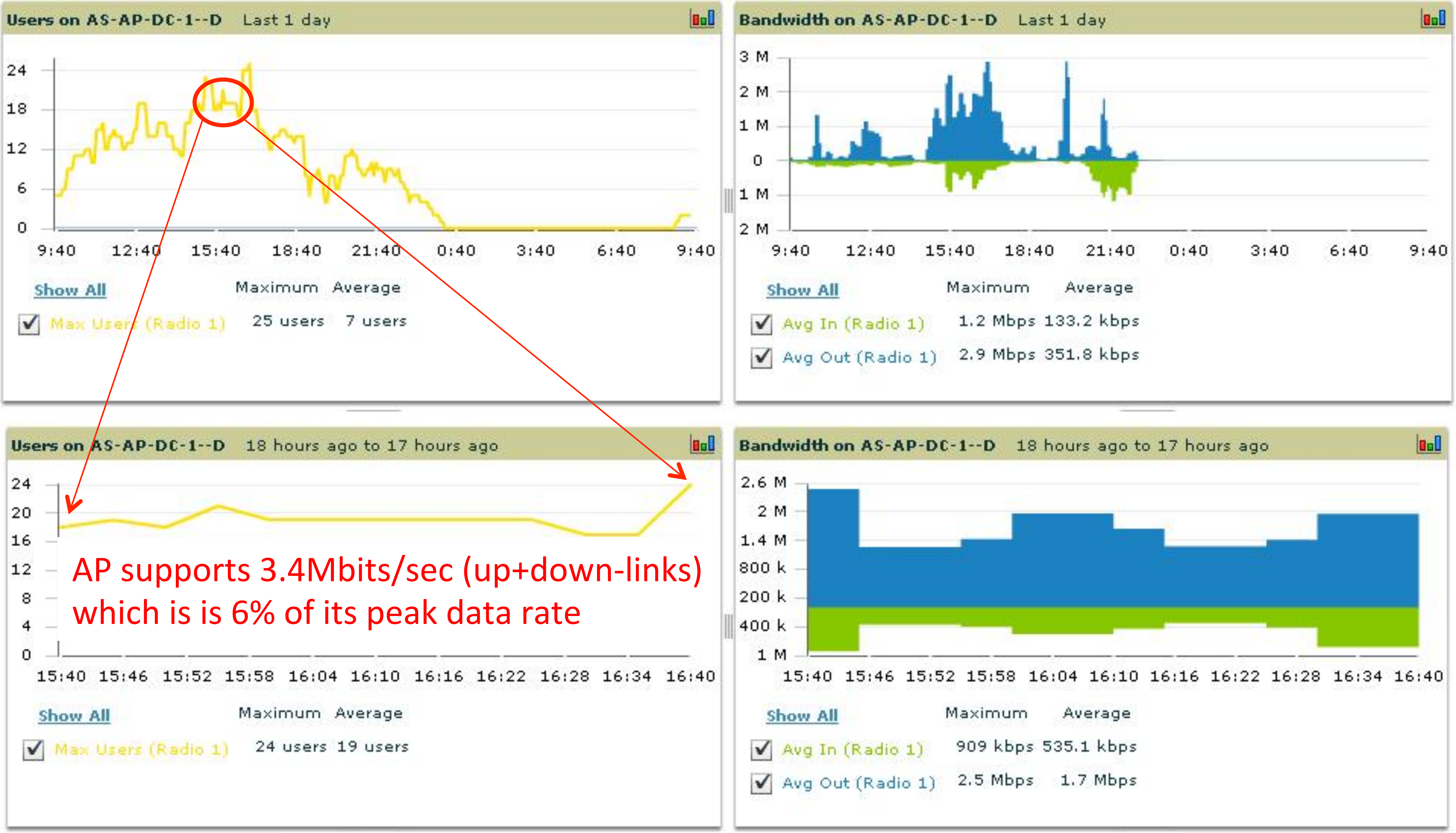


	Maximum	Average
☑ Bits Per Second In	47.9 M	20 M
☑ Bits Per Second Out	101 M	43.6 M

Wireless traffic in the entire campus

Average=70Kbits/sec/user (up+down-links)   Peak=80Kbits/sec/user (up+down-links)

A Busy Access Point in the Library



Users on AS-AP-DC-1--D Last 1 day

Show All	Maximum	Average
☑ Max Users (Radio 1)	25 users	7 users

Bandwidth on AS-AP-DC-1--D Last 1 day

Show All	Maximum	Average
☑ Avg In (Radio 1)	1.2 Mbps	133.2 kbps
☑ Avg Out (Radio 1)	2.9 Mbps	351.8 kbps

Users on AS-AP-DC-1--D 18 hours ago to 17 hours ago

Show All	Maximum	Average
☑ Max Users (Radio 1)	24 users	19 users

Bandwidth on AS-AP-DC-1--D 18 hours ago to 17 hours ago

Show All	Maximum	Average
☑ Avg In (Radio 1)	909 kbps	535.1 kbps
☑ Avg Out (Radio 1)	2.5 Mbps	1.7 Mbps

AP supports 3.4Mbits/sec (up+down-links) which is is 6% of its peak data rate

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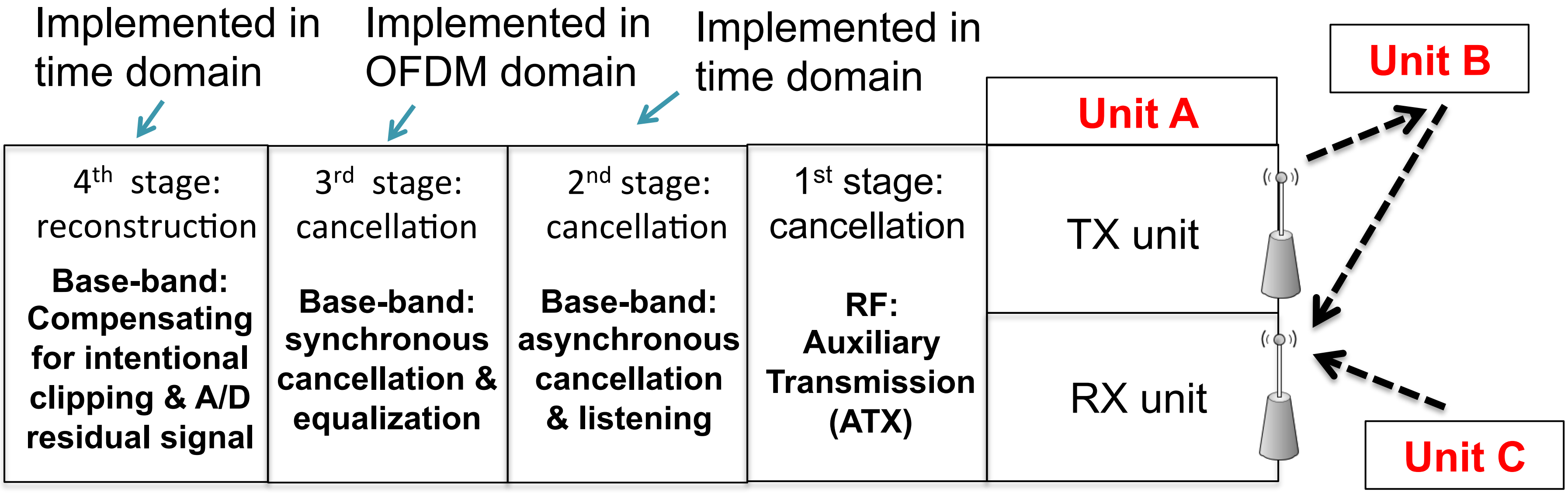
Role of Control Signaling

- Delay sensitivity of control signaling (request to join, etc.) is the primary reason for poor efficiency in WLAN, or poor quality of service.
  - Cause of shortcoming in control signaling: Each unit can either talk or listen at a given time.
  - Ideal solution: Listen (asynchronously) while Talking.

Listen (Asynchronously) while Talking



Implemented in time domain

Implemented in OFDM domain

Implemented in time domain

4th stage: reconstruction
Base-band: Compensating for intentional clipping & A/D residual signal

3rd stage: cancellation
Base-band: synchronous cancellation & equalization

2nd stage: cancellation
Base-band: asynchronous cancellation & listening

1st stage: cancellation
RF: Auxiliary Transmission (ATX)

Unit A

TX unit

RX unit

Unit B

Unit C

- Unit A is listening to detect a valid incoming signal, while transmitting to unit B.
- Made possible by the 2nd stage of self-cancellation which is synchronous with unit A's transmit signal (to cancel self interference), but asynchronous with the signal from Unit C.

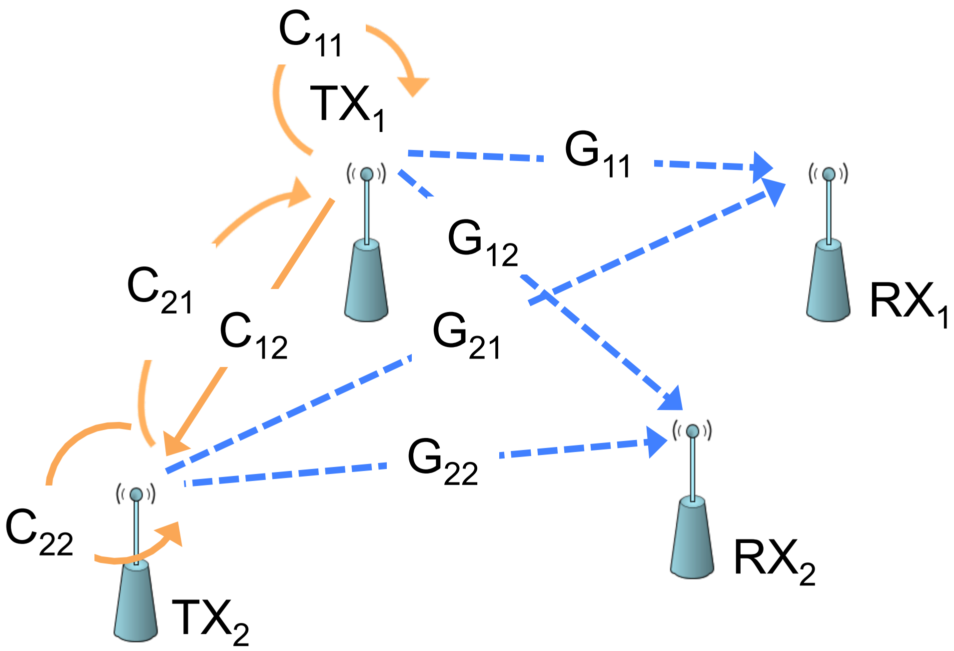
Details at www.cst.uwater.loo.ca/2way

A New Concept:

Superimposed Networking for Control Signaling

- Control signaling, particularly signaling in uplink required to join the network, is a primary bottleneck.
- Methods described earlier for handling asynchronous users enable superimposing a half-duplex, low bit rate, low power, easy to detect network for control signaling on top of the network of primary full-duplex data links.
- Features of superimposed links:
  - Separated from the primary full-duplex data links in code domain.
  - Use time multiplexing and CSMA among themselves, but conventional problems are avoided as these operate in parallel with the primary full-duplex data links.
  - Have a low spectral efficiency, but this is not an issue as control signaling has a minor load on the overall throughput.
  - PHY is designed such that full-duplex links can detect and cancel the interference caused by the superimposed control links.

Interference Channel



- R_1 and R_2 are filters at receivers of nodes TX_1 and TX_2, respectively.
- To cancel interference, we need:

$$R_1 = \frac{G_{21}}{G_{21}C_{11} - G_{11}C_{21}} \qquad R_2 = \frac{G_{12}}{G_{12}C_{22} - G_{22}C_{12}}$$

Several more cases for network application including cooperative signaling, SDMA in uplink and down-link, and Interference Channel are at: www.cst.uwaterloo.ca

A New Paradigm in Wireless:

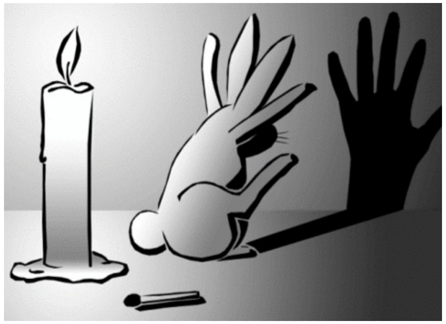
Media-based

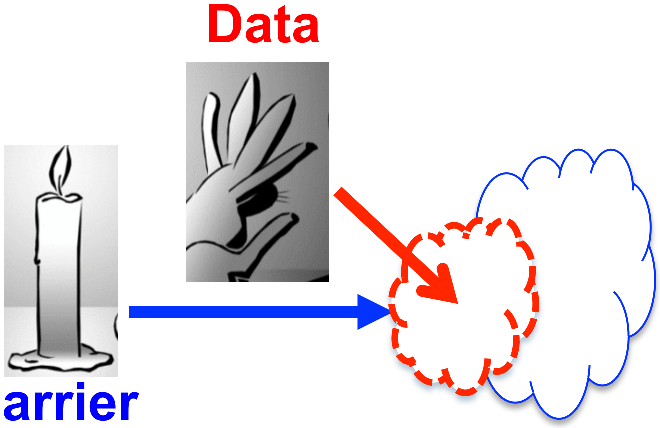
vs.

Source-based

(MADE practical using two-way)

Media-based Wireless



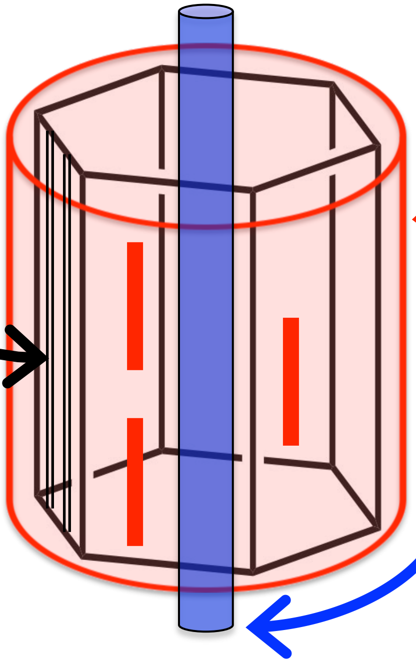


Exponential growth due to rich scattering

- Keep the source shining and change the media
- Enjoy rich variations with small changes in media
- Rich scattering environment: slightest perturbation in the environment causes independent outcomes.
- Variations of phase is critical and can be exploited with stable TX/RX synchronization using **two-way link** (continually sending back pilot from RX to TX).

How to Change the RF Channel?

Just An Example



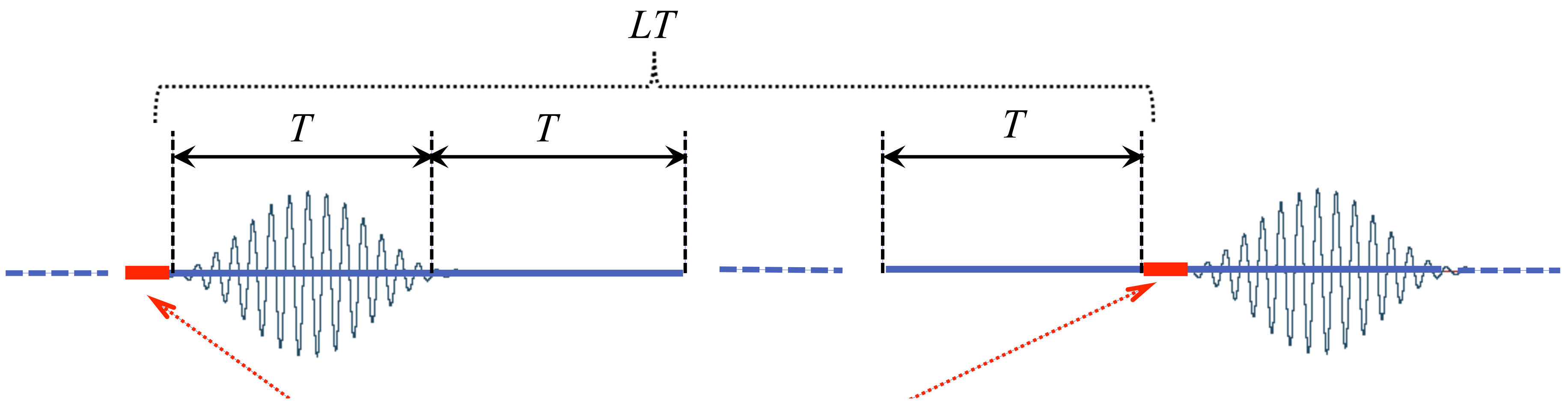
Surfaces with controllable RF properties

Enhancing rich scattering (near field reflections & openings in walls for wave to exit)

Antenna sending a Fixed/shaped carrier

- Separately control some of RF properties, i.e. μ, ε, σ of each surface, e.g., on-off partial mirrors, according to the input data (**media-based**) or randomly (**security**).
- More details at: www.cst.uwaterloo.ca/2way

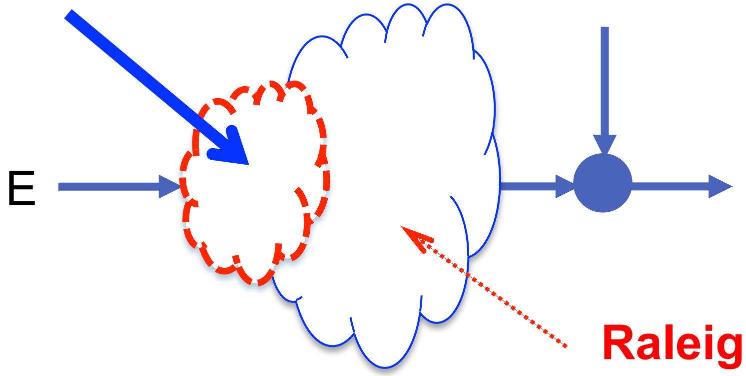
Media-based: Signaling Scheme



At these times, coded (FEC) bits are used to select one of possible channel states.

Coded Data

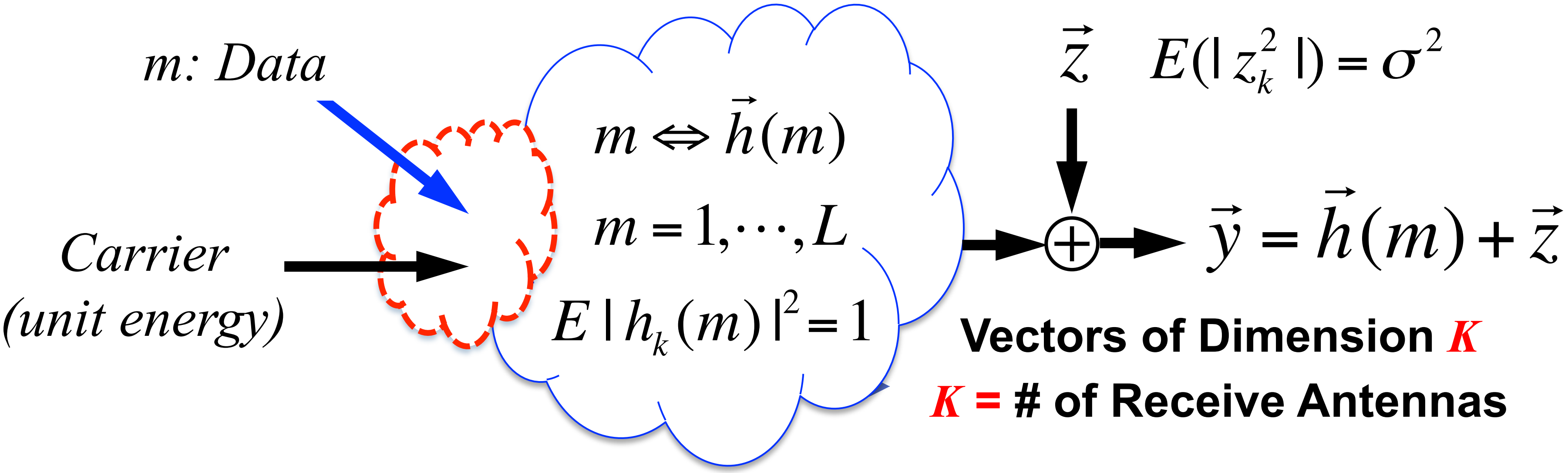
Media-based
one complex dimension



Total signal energy: LE
Basis: Orthogonal
Complex Dimensions: L

Raleigh channel of impulse response of length L.

Media-based: Rate



m: Data

Carrier (unit energy)

$m \Leftrightarrow \vec{h}(m)$

$m = 1, \cdots, L$

$E|h_k(m)|^2 = 1$

$\vec{z}$ $\quad E(|z_k^2|) = \sigma^2$

$\vec{y} = \vec{h}(m) + \vec{z}$

Vectors of Dimension *K*

***K* = # of Receive Antennas**

$$I(\vec{y};m) = I(\vec{y};\vec{h}(m)) = H(\vec{y}) - H(\vec{z}) = H(\vec{y}) - K\log_2(2\pi e\sigma^2)$$

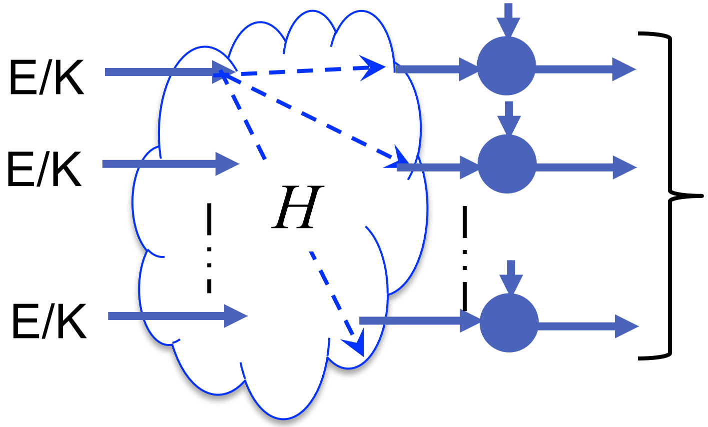
$\vec{h}(m), m = 1, \cdots, L$: K-D constellation (iid Gaussian elements)

$$H(\vec{y}) \approx \frac{1}{L}\sum_i |h_i|^2 \times \log_2(1 + 2\pi e\sigma^2)$$

For low SNR, i.e., prior to saturation to $\log_2(L)$

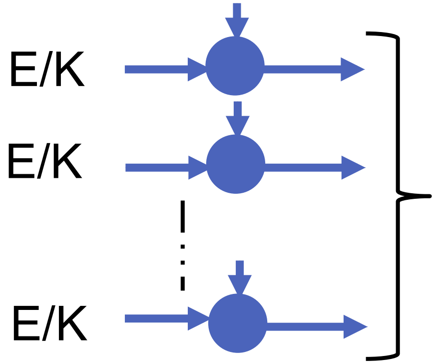
Media-based vs. Source-based

KxK
MIMO
K complex
Dimensions



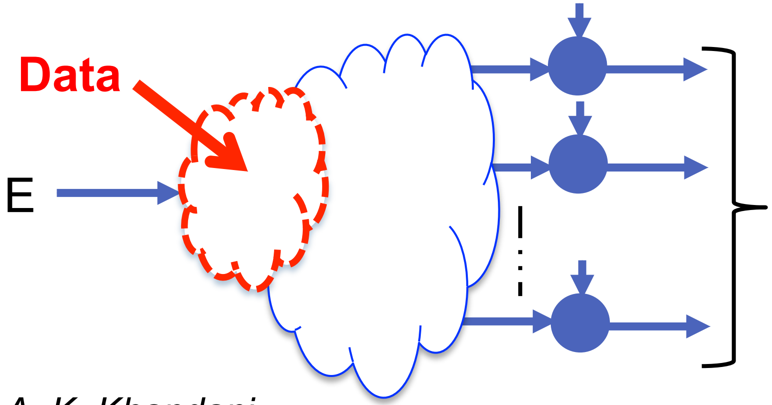
Total signal energy: E
Basis: non-orthogonal
Complex Dimensions/sec/Hz: K

Parallel
SISO
K complex
dimensions



Total signal energy: E
Basis: Orthogonal
Complex Dimensions/sec/Hz: K

Media-based
one complex
dimension



Total signal energy: LKE
Basis: Orthogonal
Complex Dimension/sec/Hz: K

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Media-based vs. Source-based (legacy)

- **Source-based**: Source is varied to embed data and then is passed through (static) channel
  - System is linear time-invariant
  - KxN MIMO: capacity scales with min(N,K), KxK MIMO:

$$C(E) \leq K \log(1+E/K)$$

with equality for K parallel AWGN SISO of gain one.

- **Media-based:** Channel is varied to embed data
  - System is linear time-varying
  - 1xK Media-based (i.i.d sampling over space):

$$C(E) = K \log(1+LE)$$

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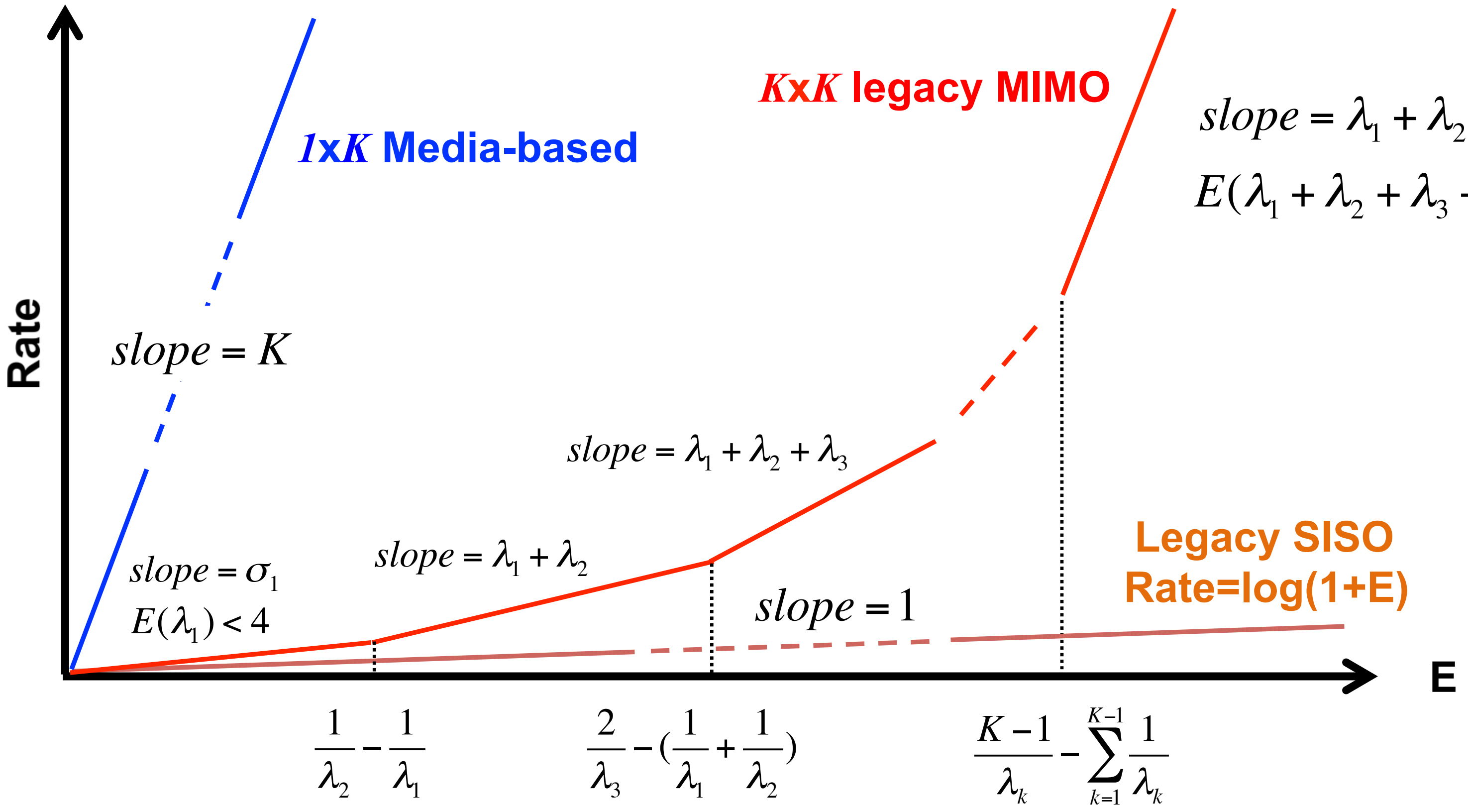
Media-based vs. Legacy Systems: Effective Dimensionality

$\lambda_1 > \lambda_2 > \ldots > \lambda_K$: Eigenvalues of a KxK Wishart random matrix



Asymptotic Gain (for E→∞) in Rate and Energy

$$\begin{cases} C_{MIMO}(E) = K\log_2\left(\frac{E}{K} + \frac{1}{K}\sum \frac{1}{\lambda_k}\right) + \sum \log_2(\lambda_k) \\ C_{Media-based}(E) = K\log_2(1 + LE) \end{cases}$$

$$E \to \infty \begin{cases} \overline{C_{MIMO}(E)} \approx K\log_2\left(\frac{E}{K}\right) + \overline{\sum \log_2(\lambda_k)} = K\log_2(E) + \overline{\sum \log_2\left(\frac{\lambda_k}{K}\right)} \\ C_{Media-based}(E) \approx K\log_2(LE) \end{cases}$$

$$\Delta E(L,K) \cong \frac{10}{K \times \log_2(10)} \overline{\sum \log_2\left(\frac{\lambda_k}{KL}\right)} \qquad \Delta R(L,K) \cong -\overline{\sum \log_2\left(\frac{\lambda_k}{KL}\right)}$$

Asymptotic Gain (for E→∞) in Energy & Rate

$$\Delta E(L,K) \cong \frac{10}{K \times \log_2(10)} \overline{\sum \log_2\left(\frac{\lambda_k}{KL}\right)}, \qquad \Delta R(L,K) \cong -\overline{\sum \log_2\left(\frac{\lambda_k}{KL}\right)}$$

K	L	KL	ΔR	ΔE
1	**2**	**2**	**4.2 bits**	**6.4 dB**
2	**1**	**2**	**4.2 bits**	**9.4 dB**
1	**4**	**4**	**13.0 bits**	**9.8 dB**
4	**1**	**4**	**13.0 bits**	**15.8 dB**
1	**8**	**8**	**34.8 bits**	**13.1 dB**
8	**1**	**8**	**34.8 bits**	**22.1 dB**
1	**16**	**16**	**86.4 bits**	**16.2 dB**
16	**1**	**16**	**86.4 bits**	**28.2 dB**

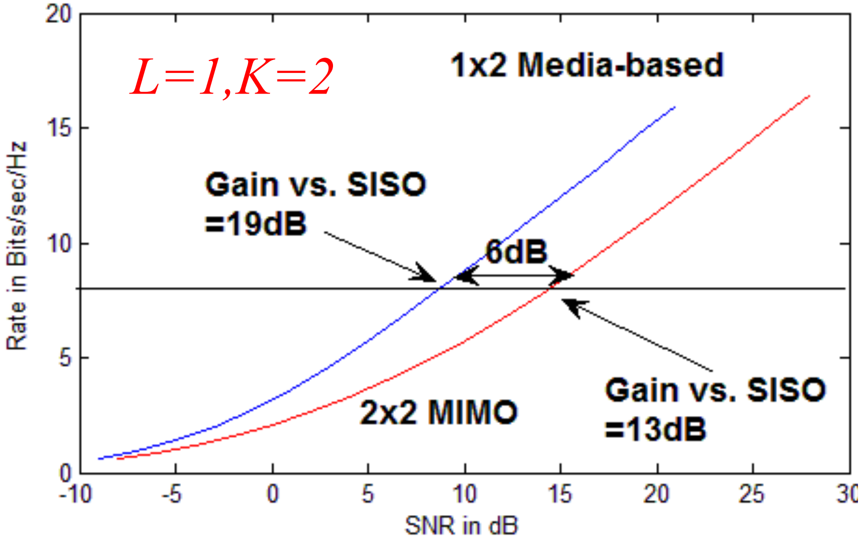
Media-based vs. Legacy Systems: Slope of Rate vs. SNR (dB) at SNR=0

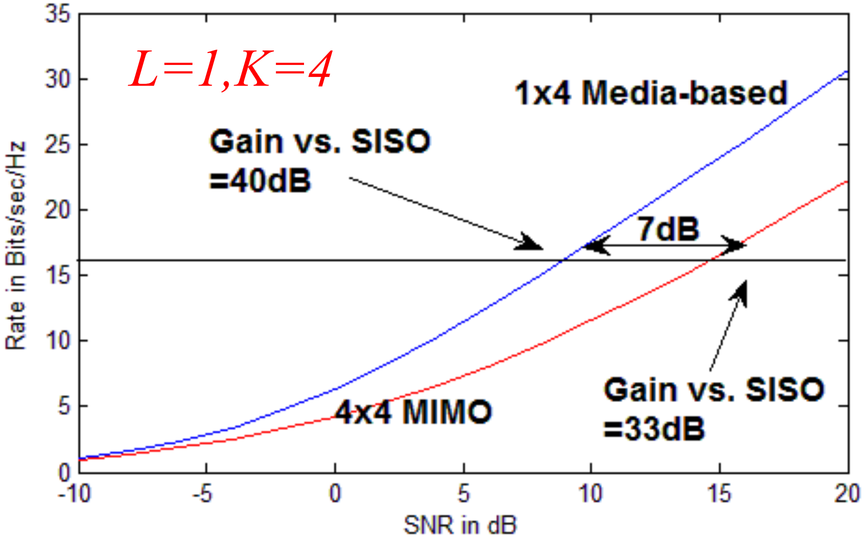
- Legacy SISO: Slope=**1**
- Legacy KxK MIMO: **Maximum eigenvalue of a KxK Wishart matrix (upper limited by 4)**
- 1xK Media-based: **LK**

	L=1, K=2	L=1, K=4	L=1, K=8	L=1, K=infinity
KxK MIMO	1.75	2.45	2.96	4
1xK Media-based	2	4	8	Infinity

Media-based vs. Legacy MIMO

2 or 4 antenna with a rate of 4 bits/s/Hz/antenna is typical in current systems



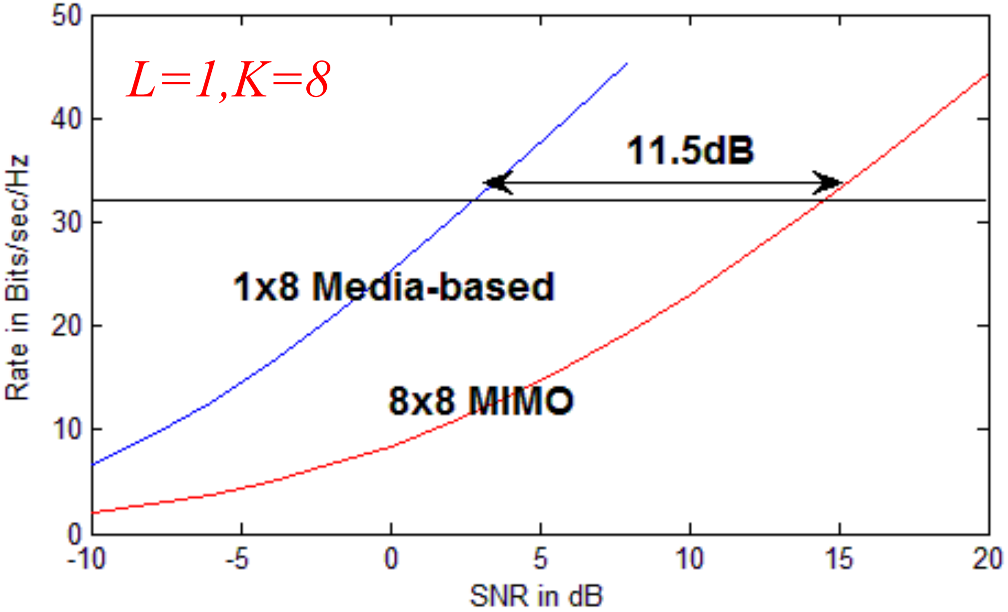


Relative Gain of Media-based will be much higher for $L>1$, typically $L>>1$

- MIMO works well only at high SNR values, but in some applications, e.g., optical transmission where MIMO is formed over different polarizations, or very lower power wireless, it is important to use spatial degrees of freedom offered by multiple antennas to save energy.

Media-based vs. Legacy MIMO

8 antenna systems with a rate of 4 bits/s/Hz/antenna



Relative Gain of Media-based will be much higher for $L>1$, usually $L>>1$.

- 1xK media-based is significantly simpler than KxK MIMO.
- **At low SNR**, unlike MIMO, media-based is optimum.
- **At high SNR**, energy saving of media-based vs. MIMO is significant and increases with the number of antennas.

Some Remarks

- Power spectrum for media-based:
  - Linear time-varying (LTV) while legacy systems are linear time-invariant (LTI).
  - LTV can cause frequency expansion.
  - Received Power Spectrum: Average of channels' spectrums times input spectrum
  - Input spectrum is shaped to limit the bandwidth.
- Equalization:
  - Channels with an impulse response of length L provides L extra dimensions per receive antenna (inserting time gaps between subsequent transmissions).

SISO Case Revisited

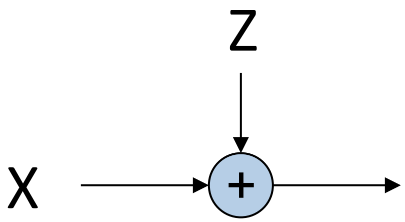
- TX block is a train of K consecutive base TX signals, followed by $L-1$ zeros prior to the next TX block.
- Channel in changed in each of K time slots among 2^r possibilities, resulting in a linear system with a random impulse response.
  - Time shift in input results in the same time shift in the response.
  - Oversample RX signal (sum of time-shifted responses) by L.
    - KL samples are full rank, yielding $LK^2/(L+K-1)$ dimensions per unit time.
    - Extra dimensions are correlated, degrading the performance.
    - Noise is correlated, improving the performance.
    - Iterative or Trellis decoding can be used for detection.
  - Source code-book is composed of a discrete set of shells (circular shells) with uniform phase.

More discussion on Media-based, including discussions on embedding information in both source and channel are at: www.cst.uwaterloo.ca

Security Applications: One-time Pad (Vernam Cipher)

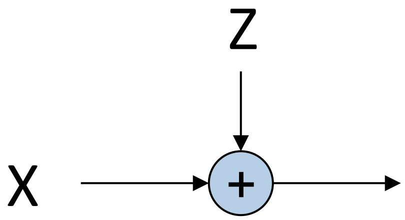
Unbreakable Security: Vernam Cipher, One-time Pad

- Vernam Cipher, One-time Pad: Bit-wise XOR of a (non-reusable) mask with the message



Y=X+Z (mod 2), X,Y,Z are binary, I(X;Y)=0

- Generalization:



Y=X+Z (mod 2π), X,Y,Z are angle, I(X;Y)=0

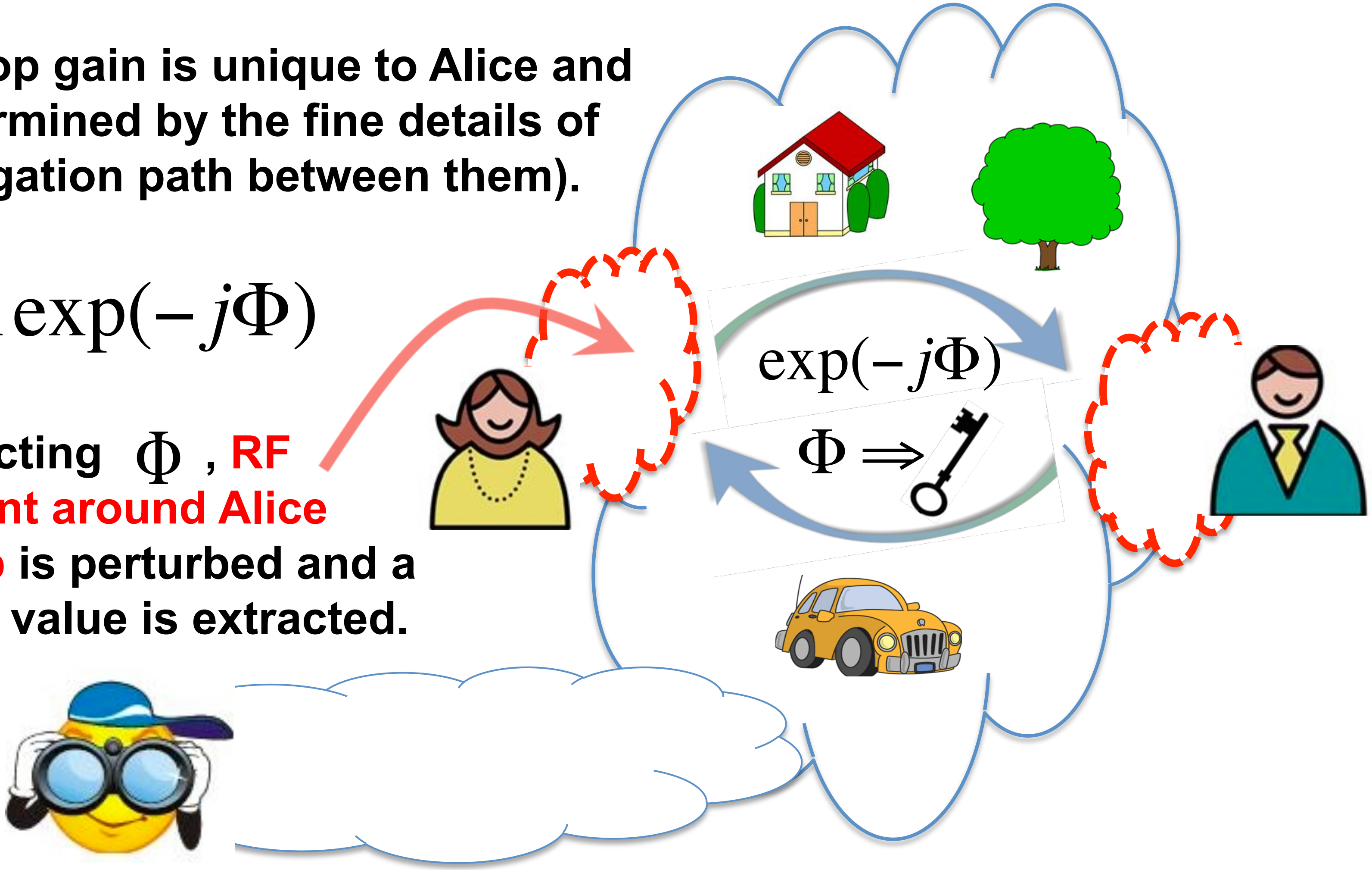
  - Happens naturally in wireless transmission
    - Each random phase can completely mask one PSK symbol.

Exploiting Channel Randomness to Share a Common Phase (Key)

Closed loop gain is unique to Alice and Bob (determined by the fine details of the propagation path between them).

$$G = A\exp(-j\Phi)$$

After extracting ϕ , RF environment around Alice and/or Bob is perturbed and a new phase value is extracted.



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Exploiting Channel Randomness to Share a Common Phase (Key)

- **Key Point:** Each TX antenna should be used only once for each channel perturbation.
- Challenges:
  - Synchronizing the two parties to agree on phase.
  - Providing a new common random phase for each PSK symbol.
  - Two-way wireless solves both these challenges.
- Errors in common phase values are corrected by the overall channel code.
- Hardware implementation shows that the common phase values can be measured with very high precision.

Exploiting Channel Randomness to Share a Common Phase (Key)

- Two algorithms are provided for key generation.
  - Please see www.cst.uwaterloo.ca/2way for details.

More discussion on security, including discussions on causing confusion using legitimate jamming, embedding information in both source and channel are at:

www.cst.uwaterloo.ca

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Thank You!

Questions/Comments?

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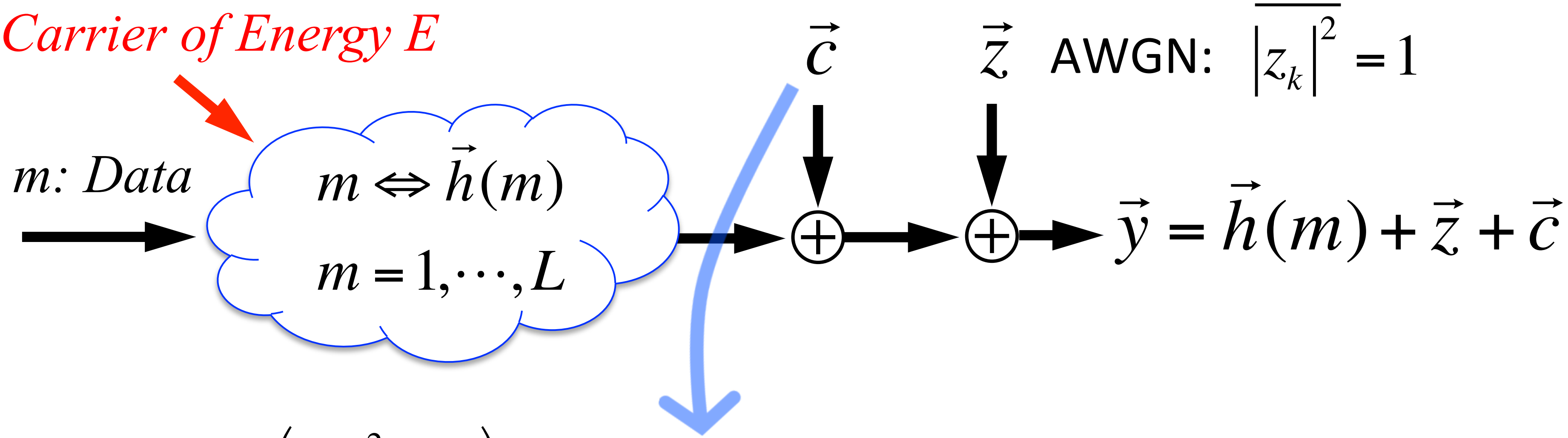
Accounting for Discrete Constellation



Carrier of Energy E

m: *Data*

$m \Leftrightarrow \vec{h}(m)$

$m = 1, \cdots, L$

$\vec{c}$

$\vec{z}$ AWGN: $\overline{|z_k|^2} = 1$

$\vec{y} = \vec{h}(m) + \vec{z} + \vec{c}$

$$P\left(\frac{\|\vec{c}\|^2}{E} > x\right) = \left(1 - \int_0^{x/2} t^{K/2-1} e^{-t} dt \,/\, \Gamma(K/2)\right)^L$$

$$\frac{\overline{\|\vec{c}\|^2}}{E} = \int_0^{\infty} \left(1 - \int_0^{x/2} t^{K/2-1} e^{-t} dt \,/\, \Gamma(K/2)\right)^L dx \sim \frac{E}{L^2}$$